MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 (TERM ID 2220)

PCL0016 – CALCULUS

(ALL Sections/Groups)

6 JULY 2023
9:00 A.M - 11:00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 4 pages including cover page with FOUR questions.

2. Attempt **ALL FOUR** questions and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1 (25 marks)

(a) Evaluate the following limits without using L'Hôpital's Rule.

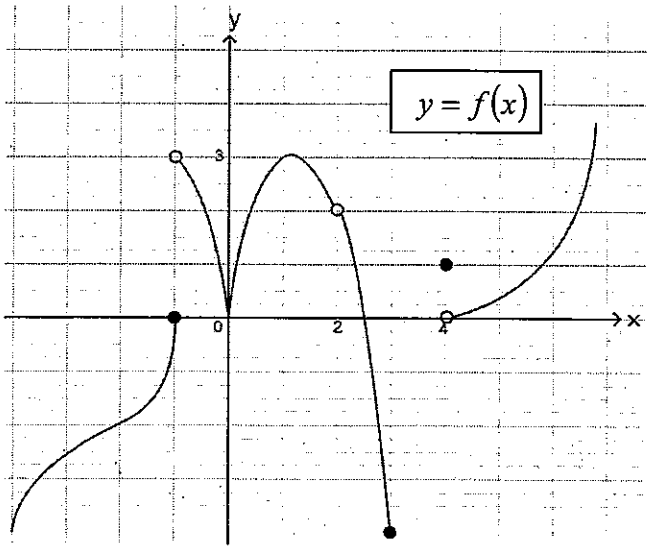
(i) $\lim_{x\to 1}\dfrac{2x^2+x-3}{x^2-1}$. [3 marks]

(ii) $\lim_{x\to -\infty}\dfrac{35x^7+20x^5-100x+11}{25x^4-12x^3+10x^2+8}$. [3 marks]

(iii) $\lim_{x\to 5}\dfrac{\sqrt{x+4}-3}{x-5}$. [4 marks]

(iv) $\lim_{x\to 0}\dfrac{\left(\sin\sqrt{2}x\right)^2}{x\tan 3x}$. [4 marks]

(b) The graph of a function $f(x)$ is given below.



Find the following limits:

(i) $\lim_{x\to -\infty} f(x)$ [1 mark]

(ii) $\lim_{x\to -1} f(x)$ [1 mark]

(iii) $\lim_{x\to 0} f(x)+\lim_{x\to 2} f(x)$ [2 marks]

(iv) $\left[\lim_{x\to -1^+} f(x)\right]\times f(4)$ [2 marks]

Continued.....

(c) From the given $f(x)$ in part (b), determine whether it is continuous at $x=1$ and at $x=4$. Explain your answers by using the conditions for continuity at a point. [5 marks]

Question 2 (25 marks)

(a) Find $\frac{d^2y}{dx^2}$ of the following:

(i) $y = x^4 - 3x^3 + 2x^2 - x - 1$ [2 marks]

(ii) $y = x\cos 2x$ [3 marks]

(b) Given $e^x y^2 = (x^2+1)^{\ln x}$, find $\frac{dy}{dx}$ by using logarithmic differentiation. [6 marks]

(c) Evaluate the following integrals:

(i) $\int (2x+1)\sin(3x)\ dx$ by using integration by parts. [5 marks]

(ii) $\int \frac{3x^2-7x-2}{x(x-1)(x+1)}dx$ by using partial fraction decomposition. [9 marks]

Question 3 (25 marks)

(a) Given a function $f(x) = 2x^3 + 9x^2 - 24x - 10$.

(i) Find the interval on which the function is increasing or decreasing. [6 marks]

(ii) Identify the local maximum or minimum of $f(x)$. [2marks]

(iii) Determine the interval(s) on which $f(x)$ is concave up or down. [4 marks]

(iv) Identify the inflection point(s) of $f(x)$. [1 mark]

Continued.....

(b) Figure 1 shows the shaded region bounded by $y=\frac{1}{4}x^2$, $x=0$ and $y=1$.

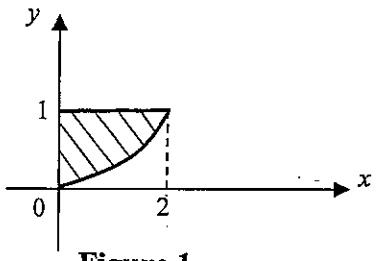


Figure 1

(i) Find the area of the shaded region. [3 marks]

(ii) Evaluate the volume of the solid obtained by rotating the shaded region about y-axis using the disk method. [4marks]

(iii) Verify the answer from part (ii) by using the cylindrical shell method. [5 marks]

Question 4 (25 marks)

(a) Solve the following first order differential equations:

(i) $\frac{dy}{dx}=\frac{x^2+2x+1}{y^2}$, $y(0)=2$. [5 marks]

(ii) $\frac{dy}{dx}+\frac{4}{x}y=\frac{e^x}{x^3}$. [7 marks]

(b) Find the solutions of the following second order differential equations.

(i) $4y''-20y'+25y=0$. [4 marks]

(ii) $\frac{1}{4}y''+16y=0, \quad y(0)=\frac{1}{3}, y'(0)=0$. [9 marks]

End of Paper